(**5 points**) Prove that ( Using Boolean expression)

**(50 points)** Simplify the function

* Using the Boolean expressions
* Using Karnaugh map
* Implement the simplified function using **NOR** and **Inverter** gates
* Implement the simplified function with a four-to-one multiplexer and external gates.
* Implement the simplified function with a 4-to-16 decoder and external gates.

**(15 points)** A digital circuit has a clock generator that produces pulses at a frequency of 8 MHz. Design a circuit that provides a clock with a frequency of 1 MHz

(**30 points**) Show the complete gate-level design of a state machine that detects the sequences of 1101 and 1110on its input. Provide a synchronous reset using external gates to start the machine in its initial state. The machine detects either sequence or overlap of the two. When the input sequence is detected, the circuit produces a complete one-clock duration pulse on its w output. Show the circuit diagram using logic gates and D flip-flops. The machine should detect the overlap of the sequence.

(**Hint**: You must consider that when you have two inputs, on each arrow of the state machine you must consider the value of both inputs. In addition, for each state there are 4 different possibilities of transitions to other states)